Bay’s Therem

DEFINITION of 'Bayes' Theorem'

A formula for determining conditional probability named after 18th-century British mathematician Thomas Bayes. This theorem provides a way to revise existing predictions or theories given new or additional evidence .It enables us to find the probability of cause In finance, Bayes' Theorem can be used to rate the risk of lending money to potential borrowers.

Also, Bayes' Theorem can be used to determine the accuracy of medical test results by taking into consideration how likely any given person is to have a disease and the general accuracy of the test

'Bayes' Theorem'

Statement

Let the events reprents a partition of sample space S. Let B be any other event defined on S. If . And then

If we write

And

Then the Bayes theorem can be stated as

Random variable:

Let E be an experiment and S be the sample space associated with it. A function X assigning to every element of S one and only one real number x = X(S) of R is called a random variable.

That is a variable used to denote the numerical value of the outcome of an experiment is called a random variable.

A random variable can be discrete or continuous depending upon the nature of its domain.

Discrete random variable: If a random variable X takes finite or countably infinite values then X is called a discrete random variable.

Continuous random variable: If a random variable X takes uncountably infinite values in a given interval then X is called a continuous random variable.

Probability Distribution of Discrete Random Variable

If is the value of X and p() is the probability of then the set of pairs is called the probability distribution.

Probability mass function (p. m. f.) or Probability density function (p. d. f.): Let X be the discrete random variable. Let be the possible values of X. with each possible outcome we associate a number p() = p() called the probability of The number p() satisfying

1. for all i
2. = 1

Then the function p is called as Probability mass function (p. m. f.) or Probability density function (p. d. f.) of random variable X.

Probability density function (p. d. f.) of continuous random variable:

A continuous function such that

1. is integrable
2. if x lies in and
3. where

is called probability density function of continuous random variable.

Remark: For discrete random variable the probability at x = c may not be zero but in continuous random variable is always zero, because

Hence for continuous random variable X

That is we may include or may not include the end points in the interval.

Mathematical Expectation of random variable

If a discrete random variable X assumes values with probabilities respectively, then the mathematical expectation of X denoted by (if it exits) and is defined as

where

If X a continuous random variable with probability density function then the mathematical expectation of X denoted by (if it exits) and is defined as

Note: 1) is expressed in the same units as X.

2) Expectation of constant is constant

i)

ii)

is denoted and is denoted by .

Laws of Expectation

Theorem 1: If X is a discrete random variable such that for all i, then

Theorem 2: If X a discrete (continuous) random variable, a and b are constants then

Theorem 3: The expectation of the sum (or difference) of two variates (discrete or continuous) is equal to the sum (or difference) of their expectations.

Theorem 3: The expectation of the product of two independent variates (discrete or continuous) is equal to the product of their expectations.

Mean and Variance of random variable

Mean

or

and

Or

Variance

i.e.

Properties of Variance:

1. Variance of constant is zero,
2. If X is a random variate and a, b are constants then

Raw and Central Moments

The following mathematical expectations have special significance in the study of probability.

r-th moment about the origin

Particular cases: =1

r-th moment about the value a

r-th moment about the mean

Particular cases:,

Moment generating Function (m.g.f.)

1. discrete random variable
2. Continuous random variable.

Moment generating Function about origin (m.g.f.)

1. Discrete random variable
2. Continuous random variable.

Some Standard distributions

Binomial Distributions

A random variable is said to follow Binomial distributions

if probability of x is given by

The two constants n and p

Re called the parameters of the distribution.

Remark:

1. The sum of the probabilities is 1
2. Let the experiment of n trials be repeated N times. Then we expect x successes to occur N. times. This is called frequency function.
3. If x is a binomial variate with parameters n and p. It is denoted as b

When do we get binomial distribution

1) A trial is repeated n times where n is a finite number.

2) Each trial results only in two ways success or failure.

3) These possibilities are mutually exclusive, exhaustive

but not necessarily equally likely.

1. If p and q are the possibilities of success and failure then

1. The events are independent. i.e. the probability p of success in each trial remains constant in all trials.

Uses:

Naturally, we can use binomial distribution when these conditions are satisfied.

In problem involving

1. the tossing of a coin- heads or tail
2. the result of an election – success or failure
3. The result of inspection of an article – defective

or non – defective.

Mean and Variance

Additive Property of Binomial Distribution

1. If is a binomial variate with parameter and and

Is another Binomial Variate with parameter and then

in general is not a Binomial variate.

1. If is a binomial variate with parameter and and is another Binomial Variate with parameter and then

is a Binomial variate with parameters

Poisson Distribution

Poisson distribution was discovered by the French Mathematician Poisson in 1837.

Poisson distribution is the limiting case of the Binomial distribution under the following conditions

1. n, the number of trails is infinitely large i.e.
2. p ,The Probability of success in each trial is constant and infinitely small i.e.
3. np, the average success is finite say m i.e. np = m.

Definition : A random variable X is said to follow Poisson distribution if the probability of x is given by

And m is called the parameter of the distribution.

Remark:

1. The sum of the probabilities is 1
2. Poisson distribution occurs where the probability of occurrence p is very small and the number of trails n is very large and where the probability of occurrence only can be known

Example:

1. Number of accidents on highway.
2. The number of deaths by a disease
3. The number of printing mistakes on a page

In these cases we can only observe the number of successes but the number of failures cannot observe.

When do we get Poisson distribution?

1. The number of trails n is infinitely large i.e.
2. A trail results in only two ways success or failure
3. If p and q are probabilities of success or failure then

p + q = 1

1. These probabilities are mutually exclusive, exhaustive but not necessarily equally likely.
2. The Probability p of success is very small
3. and such that np = m a constant.

Since Poisson distribution is the limiting case of

Binomial distribution, we can calculate Binomial probabilities approximately by using Poisson distribution whenever n is large and p is small.

Mean = variance = m

Recurrence Relation for Probabilities

This relation can be used to find expected frequencies. This is called fitting a Poisson distribution.

Normal Distribution

Normal distribution is one of the most important and commonly used continuous distribution. A large number of continuous variates follow this distribution hence the name normal

Definition:

A continuous random variable X is said to follow Normal distribution with parameter m (called mean) and (called variance) if its probability density function is given by

Remark : A continuous random variable X following normal distribution with mean m and standard deviation is referred to as

2)If X is a normal variate with parameter then is also normal variate with mean 0 and standard deviation 1. It is called standard Normal Variate.

Importance of Normal Distribution

1. The variables such as height, weight, intelligence etc. follow normal distribution.
2. Binomial and Poisson distribution can be approximated by normal distribution.
3. Many of the distribution of sample statistic e.g. Sample mean, Sample Variance tend to normal distribution for large samples.

normal distribution has wide applications in Statistical QualityControl

Let X be the continuous random variable.

continuous